

The Universality of M-branes*

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Abstract

We review the evidence for the various dualities amongst the five D=10 superstring theories and for the existence of M-theory using the associated effective supergravity theories. We also summarise the combinatorial technics developed for constructing BPS solutions in D=11 supergravity theory and conjecture that all the BPS solutions of $D < 11$ supergravity theories can be derived from the BPS solutions of D=11 supergravity that preserve 1/2 the supersymmetry. To demonstrate this, we derive the dyonic p-brane solutions from eleven dimensions.

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1 Introduction

The last two years have seen remarkable progress towards understanding the non-perturbative properties of superstring theory. Some of the main results are the dualities found amongst all five D=10 superstring theories and the evidence given for the existence of M-theory. The type IIA string compactified on a circle of radius R_A is equivalent to the type IIB string [1,2] compactified on a circle of radius R_B provided that $R_B = 1/R_A$. Similarly, the $E_8 \times E_8$ heterotic string theory compactified on a circle of radius R_E and with gauge group broken to $SO(16) \times SO(16)$ with Wilson lines is equivalent to $SO(32)$ heterotic string compactified on a circle of radius R_{SO} with gauge group again broken to $SO(16) \times SO(16)$ with Wilson lines [3,4] provided that $R_{SO} = 1/R_E$. The transformation that establishes the equivalence in both cases is T-duality which is a perturbative symmetry within the superstring theory and therefore it can be verified order by order perturbation theory. Furthermore the $SO(32)$ heterotic string is S-dual to the Type I string [5], i.e. the behaviour of $SO(32)$ heterotic string at strong string coupling is given by a weakly coupled type I string and vice-versa. Note though that equivalence of two theories under S-duality goes beyond superstring perturbation theory.

One way to establish a relation between N=1 superstrings ($E_8 \times E_8$ and $SO(32)$ heterotic, and type I) and N=2 superstrings (IIA and IIB) is via M-theory; N denotes the number of supersymmetries. (For a recent review see Schwarz [6].) There is no intrinsic definition of M-theory but assuming that its effective theory is the D=11 supergravity, it is possible to gather evidence which support the conjecture that M-theory compactified on S^1 is equivalent to the IIA string [5,7]; the compactification radius is related to the IIA string coupling constant in such a way that the strong coupling limit of IIA string is M-theory. On the other hand, compactifying M-theory on S^1/Z_2 is equivalent to the $E_8 \times E_8$ heterotic string [8]; similarly the strong coupling limit of $E_8 \times E_8$ heterotic string is again M-theory. It is clear then that all superstring theories can be derived from M-theory either as Kaluza-Klein (KK) reductions or as KK reductions followed by T- and S-duality transformations.

Evidence for the existence of a duality symmetry between two string theories can be found by examining the behaviour of the BPS part of their spectrum under the proposed duality. This is because only the BPS states are expected to be stable under the changes in the coupling constants of the two theories required by the duality. In the effective theory context, one investigates the classical solutions of the associated effective supergravity theories that preserve a proportion of the supersymmetry; we shall call these solutions BPS solutions. There are many such solutions of $D < 11$ supergravities. However, since all superstring theories in $D < 10$ dimensions are compactifications of the $D = 10$ ones and the latter are related to M-theory, all BPS solutions of the associated supergravity theories should have an M-theory interpretation.

We shall review the properties of the BPS solutions of the supergravity theories with emphasis in D=10 and D=11 dimensions and then we shall use them to explain the following relations: (i) The KK interpretation of IIA string theory as the reduction of M-theory on S^1 , (ii) the interpretation of $E_8 \times E_8$ heterotic string as the compactification of

M-theory on S^1/Z_2 , (iii) the T-duality equivalence between type IIA and IIB strings, and finally comment on the S-duality equivalence between the type I and $SO(32)$ heterotic strings. We shall then conjecture that all BPS solutions of $D < 11$ supergravity theories can be derived from the BPS solutions of D=11 supergravity theory that preserve 1/2 of the supersymmetry and we shall demonstrate this by deriving some of the dyonic p-branes in $D = 2p + 4$ from eleven dimensions.

2 p-brane combinatorics

Some of the BPS solutions of a D-dimensional supergravity theory (with maximal supersymmetry) are interpreted as p-branes, i.e. they are p-dimensional objects lying within a D-dimensional spacetime and are the sources of a $(p + 2)$ -form field strength $F_{(p+2)}$. For example the 0-branes are particles that are the sources of the Maxwell field F_2 , the 1-branes are strings that are the sources of the 3-form field strength F_3 and so on. (For a review see ref [9].) The p-brane solutions of a supergravity theory satisfy the killing spinor equations preserving 1/2 of the supersymmetry and their mass per unit volume M_p saturates the bound

$$M_p \geq \beta Q_p , \quad (1)$$

where

$$Q_p = \int_{S^{D-p-2}} \star F_{p+2} , \quad (2)$$

is the charge per unit volume and β is a positive constant. A generic p-brane solution of a D-dimensional supergravity theory is

$$\begin{aligned} ds^2 &= H^\gamma dx \cdot dx + H^\delta dy \cdot dy \\ F_{(p+2)} &= \epsilon_{p+1} \wedge dH^\rho \\ \phi &= H^\sigma , \end{aligned} \quad (3)$$

where x are the worldvolume coordinates of the p-brane spanning a $(p + 1)$ -Minkowski spacetime, y are the coordinates of the transverse space of the p-brane spanning a $(D - p - 1)$ -Euclidean space, $H = H(y)$ is a harmonic function of the transverse space, ϵ_{p+1} is the volume form of the Minkowski spacetime, ϕ are scalars and $\gamma, \delta, \rho, \sigma$ are real numbers.

Interpreting the BPS solutions of supergravity theories as extreme p-branes has been proved very successful for developing powerful combinatorial technics for constructing new BPS solutions from the p-brane solutions (3). These involve the following: (i) The magnetic dual [10, 11] of the p -brane in (3) is a \tilde{p} -brane which couples to the Poincaré dual of $F_{(p+2)}$ where $\tilde{p} = D - p - 4$. Dyonic p-branes are those for which $p = \tilde{p}$ and therefore $p = (D/2) - 2$, i.e. there are dyonic 0-branes (dyons) in D=4, dyonic strings in D=6, dyonic membranes in D=8 and dyonic 3-branes in D=10. In particular, since the Hodge star operator has real eigenvalues in $D = 6, 10$ dimensions, the possibility arises for having self-dual strings and self-dual 3-branes solutions. (ii) Any p-brane solution (3) in D-dimensions reduces to (p-k)-brane solution in $D - d$ dimensions by wrapping the worldvolume coordinates of the p-brane on the homology k-cycles of the compactifying

d-dimensional space. For toroidal compactifications this can be done explicitly since wrapping is just the KK reduction of the solution along its worldvolume directions. (iii) Given p_i -branes, $i = 1, \dots, n$, solutions of a D-dimensional supergravity theory preserving $1/2$ of the supersymmetry, a new solution can be constructed preserving $1/2^n$ of the supersymmetry which can be interpreted as the intersection of these branes on a common k-brane denoted with $(k|p_1, \dots, p_n)$. The existence of such solution is determined with certain rules [12,13] called ‘intersection rules’ and the new solution is constructed by superposing the p_i -brane, $i = 1, \dots, n$, solutions using the ‘harmonic function rule’ [14,15]. The latter rule involves the introduction of a harmonic function H_i for each p_i -brane and the solution is determined by the observation that if all but one of the harmonic functions is set to one, say H_k , the solution should reduce that of p_k -brane. (iv) Given a p -brane solution or a $(k|p_1, \dots, p_n)$ solution, we can find new solutions by either boosting ($k \geq 1$) or superposing them with a KK monopole.

3 Applications to superstrings

The p-brane solutions of D=11 supergravity are the membrane [16] and its magnetic dual the fivebrane [17] each preserving $1/2$ of the supersymmetry. We shall refer to them collectively as M-branes. Apart from the M-brane solutions of D=11 supergravity, there are many other BPS solutions that are intersections of M-branes. The M-brane intersection rules [12] are the following: 1. two membranes can intersect on a 0-brane and two fivebranes can intersect on a 3-brane, 2. a membrane and a fivebrane can intersect on a string and 3. two fivebranes can intersect on a string. Apart from the intersecting M-brane solutions constructed using these intersection rules there is another solution in D=11 that has the interpretation of a membrane within a fivebrane, $(2|2,5)$, and preserves $1/2$ of the supersymmetry [18]; we shall investigate this solution in the next section in connection with the dyonic membranes.

In superstring theory, we can classify the various p-branes according to the dependence of their mass per unit volume, M_p , from the string coupling constant λ_s as fundamental p-branes, Dirichlet p-branes or D-p-branes, and solitonic p-branes. The only fundamental extended objects in string perturbation theory are the strings with $M_1 \sim 1$, and the solitonic objects are their magnetic duals the 5-branes with $M_5 \sim \lambda_s^{-2}$. An intermediate case is the D-p-branes for which $M_p \sim \lambda_s^{-1}$. (For a review on D-branes from the string point of view see Polchinski et al [19] and for some novel properties [20].) The D=10 superstring theories have the following p-brane solutions: (i) The heterotic string contains a fundamental string and a solitonic 5-brane. (ii) The type I string contain a D-string and a D-5-brane. (iii) The type IIA string has a fundamental string and a solitonic 5-brane as well as D-p-branes for $p = 0, 2, 6, 8$ and (iv) the type IIB string has a fundamental string, a solitonic 5-brane and D-p-branes for $p = -1, 1, 3, 5, 7, 9$. (The D-p-branes for $p = -1$ and $p = 9$ are the gravitational instantons and the D=10 Minkowski spacetime, respectively.)

The KK ansatz for compactifying D=11 supergravity on a circle to D=10 are

$$\begin{aligned} ds_{(11)}^2 &= e^{-\frac{2}{3}\phi} ds_{(10)}^2 + e^{\frac{4}{3}\phi} (dx_{11} + A)^2 \\ G &= F_4 + F_3 \wedge dx_{11} , \end{aligned} \tag{4}$$

where G is the D=11 4-form field strength, the D=10 metric $ds_{(10)}^2$ is in the string frame, ϕ is the dilaton, A is the KK-vector and the rest of the notation is self-explanatory. Since $\lambda_s = \exp \langle \phi \rangle$, the radius of compactification is $R = \lambda_s^{2/3}$. The M-theory interpretation of the IIA p-branes [5, 7] is as follows: the IIA 0-branes are simply associated with the KK states of the compactification of M-theory on S^1 , the IIA fundamental string is the wrapping of D=11 membrane on S^1 , the IIA membrane is just the direct reduction of the D=11 membrane on S^1 , the IIA 4-brane is the wrapping of the D=11 fivebrane on S^1 , the IIA 5-brane is the direct reduction of the D=11 fivebrane on S^1 and the D=10 6-branes are the KK monopoles of the compactification. (With the term ‘direct reduction’ of a p-brane we mean the wrapping of the p-brane on the 0-cycle of the compactifying space.) Finally the 8-brane which is a solution of the massive IIA supergravity theory does not seem to have a direct D=11 interpretation.

As we have mentioned in the introduction, type IIA and type IIB strings are equivalent under T-duality. T-duality acts on the various p-brane solutions of the associated effective supergravity theories as follows: the fundamental IIB string and solitonic IIB 5-brane transform under T-duality to the fundamental IIA string and to the solitonic IIA 5-brane, respectively. The D-p-branes of either IIA or IIB supergravity transform under T-duality to D-(p \pm 1)-branes where the sign depends on the choice of performing the T-duality transformation on a transverse or a worldvolume directions of the D-p-brane, respectively. Thus, T-duality transforms all IIB p-brane solutions to IIA p-brane solutions and vice-versa. This serves as evidence for the equivalence of type IIA and type IIB strings. Alternatively, the equivalence of IIA and IIB can be used to derive the IIB p-branes from the M-branes by first reducing the latter to the IIA p-branes and then use the T-duality transformation to transform them to the IIB p-branes.

So far we have used the various p-brane solutions preserving 1/2 of supersymmetry to provide evidence for M-theory/IIA string and IIA/IIB strings dualities. However the BPS solutions with the interpretation of intersecting p-branes can also be used for the same purpose [21]. For example the (0|0, 1) and (0|0, 4) solutions in IIA supergravity each preserving 1/4 of supersymmetry have being interpreted as KK modes on the string and on the 4-brane, respectively. (For the (0|0, 4) solution see also ref [22].) Such solutions are expected because as we have seen, the IIA string and IIA 4-brane are derived from the wrapping of D=11 membrane and fivebrane on S^1 , and therefore from the IIA perspective there must be KK modes on the string and the 4-brane.

As in the case of IIA string, the strong coupling limit of $E_8 \times E_8$ heterotic string is also the M-theory [8]. More precisely, M-theory compactified on the orbifold S^1/Z_2 gives the $E_8 \times E_8$ heterotic string where Z_2 acts on the compactified coordinate x_{11} as $x_{11} \rightarrow -x_{11}$ and on the 4-form field strength as $G \rightarrow -G$. The compactified D=11 spacetime has as boundary the disjoint union of two D=10 spacetimes one at each fixed point of the Z_2 action on x_{11} ; the radius R of compactification is proportional to the distance between

the two fixed points and $R = \lambda_s^{2/3}$. The D=11 membrane stretches between the two D=10 sheets of the boundary of the D=11 spacetime and intersects them at a string. For small string coupling constant the two D=10 spacetimes come close together and they become the D=10 dimensional spacetime of heterotic string. On the other hand the D=11 fivebrane lies entirely within the boundary of the D=11 spacetime and in the small string coupling constant limit becomes the heterotic 5-brane. The string and 5-brane are the only stable solutions of the $E_8 \times E_8$ heterotic string effective theory. The rest of the p-branes that appear in the IIA theory are unstable in the heterotic case because the orbifold symmetry Z_2 projects out the associated form field strengths and therefore no such supersymmetric configurations can exist. The $E_8 \times E_8$ gauge group of the heterotic string arises from an ‘anomaly’ argument; we attach an E_8 gauge group to each D=10 sheet of the boundary of D=11 spacetime so for weak string coupling we get the $E_8 \times E_8$ gauge group on the D=10 spacetime which is necessary for the cancellation of anomalies of the heterotic string.

Finally, it remains to examine the duality between the two heterotic strings, and the $SO(32)$ heterotic string and the type I string. In the first case T-duality transforms the fundamental string and solitonic 5-brane of one theory to the fundamental string and solitonic 5-brane of the other. In the second case the fundamental string and solitonic 5-brane of the $SO(32)$ heterotic string under S-duality transform to the D-1-brane and D-5-brane of the type I string and vice-versa [24, 25].

4 Dyonic p-branes from M-branes

In the last two sections, we have derived some of the BPS solutions of D=10 supergravities from the M-brane solutions of D=11 supergravity, we shall now turn to do the same for the dyonic p-branes of $D = (2p + 4)$ -supergravity theories. As we have mentioned in the previous section the self-dual 3-brane [23] of IIB theory can be derived by either the D=11 membrane or fivebrane. In the former case we compactify the D=11 supergravity on S^1 and directly reduce the D=11 membrane to the IIA 2-brane. Then we use T-duality to transform the IIA 2-brane to the IIB self-dual 3-brane. Alternatively, we also compactify D=11 supergravity on S^1 but this time we wrap the D=11 fivebrane on S^1 to the IIA 4-brane. Then again we use T-duality to transform the IIA 4-brane to the IIB self-dual 3-brane. Next the dyonic membrane solutions of D=8 N=2 supergravity can be obtained from M-theory in two different ways. First we start from the $(2|2, 5)$ solution of D=11 supergravity which has the interpretation of a membrane within a fivebrane [18]. We then compactify the D=11 supergravity on T^3 wrapping $(2|2, 5)$ along the three directions of the fivebrane orthogonal to the membrane to get the dyonic membrane solutions of N=2 D=8 supergravity. Alternatively, we compactify IIB on T^2 and wrap the IIB self-dual 3-brane on the linear combination

$$n\alpha_1 + m\alpha_2 , \tag{5}$$

of the fundamental homology cycles α_1, α_2 of T^2 where (n, m) are integers. It turns out that the self-dual 3-brane solution of IIB reduces the dyonic membrane solution D=8

N=2 supergravity [26] with charge (n, m) . Next, the self-dual string [23] of D=6 N=4 supergravity can be obtained by compactifying the IIB supergravity on K_3 and wrapping the self-dual 3-brane on the homology 2-cycles [27] of K_3 . Another way to derive the self dual string is to begin from the $(1|2, 5)$ solution of D=11 supergravity with the harmonic function associated with the membrane identified with the harmonic function associated with the fivebrane and perform the following chain of KK reductions and T-duality transformations:

$$(1|2, 5)_M \xrightarrow{S^1} (1|2, 4) \xrightarrow{T} (1|3, 3)_B \xrightarrow{T^4} 1^+ , \quad (6)$$

where $(1|3, 3)_B$ is a solutions of IIB supergravity and in the last step we have wrapped the four worldvolume coordinates of the two 3-branes which are orthogonal to the string on a linear combination of homology 2-cycles of T^4 . Finally the dyons of D=4 N=4 supergravity can be derived either by compactifying the D=8 N=2 supergravity on K_3 and by wrapping the dyonic membrane solutions on the homology 2-cycles of K_3 or by compactifying the D=6 N=4 supergravity on T^2 and by wrapping the self dual string on the linear combination (6) of the fundamental 1-cycles of 2-torus [28].

5 Summary

The investigation of the strong coupling limit of type IIA and $E_8 \times E_8$ heterotic strings has naturally led to M-theory which has as an effective theory the D=11 supergravity. There is no intrinsic definition of M-theory but the hypothesis that such theory exists has been proved very useful to find a link between the N=1 and N=2 D=10 superstrings and to develop a systematic way to construct the BPS solutions of $D < 11$ supergravity theories from the BPS solutions of D=11 supergravity which preserve 1/2 of the supersymmetry together with three rules that determine their allowed intersections.

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